Roll No. | | | | | | |  Date:

# NORTHWEST ACCREDITATION COMMISSION, USA

## HIGH SCHOOL DIPLOMA (Sr. Secondary/12TH)

**Subject- Mathematics**
**Subject Code – M405**

Question Paper No. : | | | | |  Question Paper code: | | | |

### Important Instructions

1. **OPENING AND CHECKING OF THE QUESTION-BOOKLET**
   Break open the seal of the Question-Booklet only when the announcement is made by the Invigilator. After breaking the seal and before attempting the questions, student should immediately check for:
   a) The number of the printed page in the Question-Booklet is the same as mentioned on the cover page of the Booklet and
   b) Any printing error in the Booklet pages, if any.
   Any discrepancy or error should be brought to the notice of the Invigilator who will then replace the Booklet. No additional time will be given for this.

2. No student, without the permission of the Superintendent, or the Invigilator concerned, is to leave his/her seat or the Examination Room.

3. **FILLING UP THE REQUIRED INFORMATION ON QUESTION-BOOKLET AND ANSWER SHEET**
   After breaking open the seal and checking the Booklet, student should:
   a) Fill up the **Question Paper No.** and **Question Paper Code** (mentioned on the cover of Question-Booklet) in the space provided on the First Answer Sheet.
   b) Fill up his/her Roll Number on the First Answer Sheet and on each Supplementary Answer Sheet, if taken.
   c) Student should mention the total number of **Supplementary Answer Sheet**, if taken, in the space provided on the First Answer Sheet and also fill up the Serial Number mentioned on each **Supplementary Answer Sheet** along with his/her Roll Number in the register maintained by the Invigilator. Student must tie all the Answer Sheets with the thread provided by the Invigilator.

4. **INSTRUCTIONS ABOUT QUESTION PAPER**
   This Question Paper is divided into three Sections **– A**, **B** and **C**. All Sections are compulsory. Attempt all Sections as per instructions.
   a) Section A question No. 1 to 10 are very short questions carrying 2 marks each.
   b) Section B question No. 11 to 25 are short questions carrying 3 marks each.
   c) Section C question No. 26 to 32 are long questions carrying 5 marks each.

5. Student found in possession of Cellular Phone / Mobile Phone / Pager or any other Communication Device and/or any Book/Note whether using or not, will be liable to be debarred for taking examination(s) either permanently or for specified period or/and dealt with as per law or/and ordinance of the School/SERI according to the nature of offence, or/and he/she may be proceeded against and shall be liable for prosecution under the relevant provision of the Statutory Law.

**TIME: 3 Hours.** **TOTAL MARKS: 100**

**THE ANSWER SHEET IS TO BE RETURNED ON COMPLETION OF THE TEST**



**P.T.O.**

## SECTION A

| Total number of questions: 10 | Marks allocated to each question: 2 | Total marks: 20 |
|---|---|---|

Question 1. Write the differential equation obtained by eliminating the arbitrary constant C in the equation representing the family of curves $xy = C\cos x$.

Question 2. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$.

Question 3. Evaluate $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Question 4. Find the sine of the angle between the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z - 5 = 0$.

Question 5. Using principal value, evaluate: $Sin^{-1}\left(Sin\frac{3\pi}{5}\right)$

Question 6. Find the equation of the line parallel to x axis and passing through the origin.

Question 7. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find (x – y).

Question 8. Write the vector equation of the following line:

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$$

Question 9. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

Question 10. Evaluate: $\int_{-1}^{1} f(x)dx$ where f(x) = x-[x]; [x] is the integral part of x.

## SECTION B

| Total number of questions: 15 | Marks allocated to each question: 3 | Total marks: 45 |
|---|---|---|

Question 11. Evaluate: $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x}\,dx$

Question 12. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes. Hence find the mean of the distribution.

Question 13. Show that the function f defined as follows, is continuous at x = 2, but not differentiable there at:

$$f(x) = \begin{cases} 3x-2, & 0 < x \le 1 \\ 2x^2 - x, & 1 < x \le 2 \\ 5x-4, & x > 2 \end{cases}$$

Question 14. Solve the following differential equation:

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$$

Question 15. Solve: $tan^{-1}(x+1) + tan^{-1}(x-1) = tan^{-1}\frac{8}{31}$, x>0

P.T.O.

Question 16. Using properties of determinants, prove the following:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

Question 17. Solve the differential equation: $\frac{dy}{dx} - \frac{y}{x} = 2x^2$.

Question 18. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angle if $k^2 = 8$.

Question 19. Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$

Question 20. Prove that $[\vec{a}, \vec{b}+\vec{c}, \vec{d}] = [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$

Question 21. Differentitate $\sin(x^2 + 1)$ with respect to *x* from first principle.

Question 22. Using vectors, prove that the line segment joining the mid-point of non-parallel sides of a trapezium is parallel to the base and is equal to half the sum of the parallel sides.

Question 23. A boy throws a ball vertically upwards from the ground to touch the top of a pole and fall down. If the ball falls down on the ground in just 4 seconds, find the height of the pole. (Use $g = 10$ m/sec$^2$)

Question 24. Evaluate: $\int_0^4 (|x| + |x-2| + |x-4|)dx$

Question 25. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing
(b) srictly decreasing

## SECTION C

| Total number of questions: 7 | Marks allocated to each question: 5 | Total marks: 35 |
|---|---|---|

Question 26. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760.00 to invest and has space for at most 20 items. A electronic sewing machine costs him Rs.360.00 and a manually operated sewing machine Rs. 240.00. He can sell an Electronic Sewing Machine at a profit of Rs. 22.00 and a manually operated sewing machine at a profit of Rs.18.00. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize his profit. Make it as a linear programming problem and solve it graphically. Keeping the rural background in mind justify the 'values' to be promoted for the selection of the manually operated machine.

Question 27. Evaluate: $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$

OR

Evaluate: $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Question 28. There are two bags **I** and **II**. Bag **I** contains 2 white and 3 red balls and Bag **II** contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag **II**.

**P.T.O.**

OR

8% of people in a group are left handed. What is the probability that 2 or more of a random sample of 25 from the group are left handed?

Question 29. If an old man rides his motor cycle at 25 km/hr, he has to speend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases to Rs.5 per km. He has Rs 100 to spend on petrol and wishes to find maximum distance he can travel within one hour.Express this as a linear programming problem and then solve it.

OR

A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that be can sell all the items that he can buy, how should he invest his money in order to maximise the profit? Formulate this as a linear programming problem and solve it graphically.

Question 30. Using matrices, solve the following system of linear equations:

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

Question 31. A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.

Question 32. Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

OR

Find the area of the region in the first quadrant enclosed by the y-axis, the line y = x and the circle $x^2 + y^2 = 32$, using integration.



**END OF THE QUESTION PAPER**