Roll No.: | | | | | | | | | | |  Date: | | | | | | |

# NORTHWEST ACCREDITATION COMMISSION, USA
# HIGH SCHOOL DIPLOMA (Sr. Secondary/12TH) 2018-2019

**Subject- MATHEMATICS** | **Question Paper No. : M T 4 5**

**Subject Code : ME1207** | **Question Paper Code: M S 7 1**

**Total Time: 03.00 Hours.** | **Total Marks: 100**

## GENERAL INSTRUCTIONS

**1. OPENING AND CHECKING OF THE QUESTION-BOOKLET**

Break open the seal of the Question-Booklet only when the announcement is made by the Invigilator. After breaking the seal and before attempting the questions, student should immediately check for:

a) The number of the printed page in the Question-Booklet is the same as mentioned on the cover page of the Booklet and

b) Any printing error in the Booklet pages, if any.

Any discrepancy or error should be brought to the notice of the Invigilator who will then replace the Booklet. No additional time will be given for this.

**2**. No student, without the permission of the Superintendent or the Invigilator concerned, is to leave his/ her seat or the Examination Room.

**3. FILLING UP THE REQUIRED INFORMATION ON QUESTION-BOOKLET AND ANSWER SHEET**

After breaking open the seal and checking the Booklet, student should:

a) Fill up the **Question Paper No.** and **Question Paper Code** (mentioned on the cover of Question-Booklet) in the space provided on the First Answer Sheet.

b) Fill up his/her Roll Number on the First Answer Sheet and on each Supplementary Answer Sheet, if taken.

c) Student should mention the total number of **Supplementary Answer Sheet**, if taken, in the space provided on the First Answer Sheet and also fill up the Serial Number mentioned on each **Supplementary Answer Sheet** along with his/her Roll Number in the register maintained by the Invigilator. Student must tie all the Answer Sheets with the thread provided by the Invigilator.

**4. INSTRUCTIONS ABOUT QUESTION PAPER**

**This Question Paper is divided into three Sections – A, B and C. All Sections are compulsory. Attempt all Sections as per instructions.**

**a) Section A contains 8 questions which are very short carrying 3 marks each.**

**b) Section B contains 10 questions which are short carrying 4 marks each.**

**c) Section C contains 6 questions which are long carrying 6 marks each.**

**5. Student found in possession of Cellular Phone / Mobile Phone / Pager or any other Communication Device and/or any Book/Note whether using or not, will be liable to be debarred for taking examination(s) either permanently or for specified period or/and dealt with as per law or/and ordinance of the School/SERI according to the nature of offence, or/and he/she may be proceeded against and shall be liable for prosecution under the relevant provision of the Statutory Law.**

**THE ANSWER SHEET IS TO BE RETURNED ON COMPLETION OF THE TEST**

This Question Paper **MUST** be attached with Answer Sheet

## SECTION A

| Total number of questions: 8 | Marks allocated to each question: 3 | Total marks: 24 |
|---|---|---|

**Question 1.** Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$ .

**Question 2.** Let E={1,2,3,4} and F={1,2}. Then find the number of onto functions from E to F.

**Question 3.** Evaluate: $\int \frac{1+\cot x}{x+\log \sin x} dx$

**Question 4.** For what value of k, the matrix $\begin{pmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{pmatrix}$ is skew symmetric?

**Question 5.** Find a unit vector in the direction of $2\hat{i} - \hat{j} + 2\hat{k}$

**Question 6.** . Write the value of the determinant:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

**Question 7.** Evaluate : $\int \sqrt{4x - x^2} dx$.

**Question 8.** If R = {(x, y): x + 2y = 8} is a relation on N, write the range of R.

## SECTION B

| Total number of questions: 10 | Marks allocated to each question: 4 | Total marks: 40 |
|---|---|---|

**Question 9.** Show that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

**Question 10.** There are 5 cards numbered 1 to 5. One number on one card. Two cards are drawn at random without replacement. Find the probability distribution of the sum of the numbers on the two cards.

**Question 11.** Evaluate: $\int_0^{\pi/4} \log(1+\tan x)\, dx$

**Question 12.** Prove that $[\vec{a}, \vec{b}+\vec{c}, \vec{d}] = [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$

**Question 13.** The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of (a) the perimeter, (b) the area of the rectangle.

**Question 14.** Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

**Question 15.** Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

**Question 16.** Evaluate: $\int \frac{5x}{(x+1)(x^2+9)} dx$

**Question 17.** A ball projected with a velocity of 28 m/sec has a horizontal range 40 m. Find the two angles of projection.

**Question 18.** Find the equation of the sphere which passing through the points (3, 0, 0), (0, -1, 0), (0, 0, -2) and having the centre on the plane $3x + 2y + 4z = 1$.

## SECTION C

| Total number of questions: 6 | Marks allocated to each question: 6 | Total marks: 36 |
|---|---|---|

**Question 19.** Bag A contains 3 red and 4 black balls and bag B contains 4 red and 5 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

**OR**

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/3, what is the probability that the student knows the answer given that he answered it correctly?

**Question 20.** Find the area of the region in the first quadrant enclosed by the y-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$, using integration.

**OR**

Find the vector and Cartesian equations of the plane containing the two lines

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \text{ and } \vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$$

**Question 21.** Evaluate: $\int_0^{\pi} \frac{x\sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi} dx$

**OR**

Evaluate: $\int \frac{1}{\sin^4 x + \sin^2 x\cos^2 x + \cos^4 x} dx$

**Question 22.** A company manufactures two types of sweaters, type A and type B. It costs Rs. 360 to make one unit of type A and Rs. 120 to make a unit of type B. The company can make atmost 300 sweaters and can spend atmost Rs. 72000 a day. The number of sweaters of type A cannot exceed the number of type B by more than 100. The company makes a profit of Rs. 200 on each unit of type A but considering the difficulties of a common man the company charges a nominal profit of Rs. 20 on a unit of type B. Using LPP, solve the problem for maximum profit.

**OR**

If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum, when the angle between them is 60°.

**Question 23.** Find the inverse of fhe following matrix using elementary operations:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

**OR**

Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

**Question 24.** A dealer in rural area wishes to purchase a number of sewing machines. He has only rupees 5760.00 to invest and has space for at most 20 items. Electronic sewing machines cost him rupees 360 and manually operated sewing machine Rs.240. He can sell an electronic sewing machine at a profit of rupees 22 and a manually operated sewing machine at a profit of Rs.18. Assuming that he can sell all the items he can buy, how should he invest his money in order to maximize his profit. Make it as a Linear Programming Problem and solve it graphically. Justify the values promoted for the selection of the manually operated machines.

**OR**

A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.

**END OF THE QUESTION PAPER**